Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Thursday 08 October 2020

Afternoon (Time: 1 hour 30 minutes)	Paper Reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have: Mathematical Formulae and Statistical Tables (Lilac), calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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P 6 5 7 5 8 R A 0 1 3 2

Pearson

1. Solve, for $0 \leqslant x < 360°$, the equation

$$2\cos 2x = 7\cos x$$

giving your solutions to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

1. $2\cos(2x) = 7\cos(x)$

$\boxed{\text{USING DOUBLE ANGLE FORMULAE} \rightarrow \cos(2A) = \cos^2(A) - \sin^2(A) = \cos^2(A) - (1-\cos^2(A)) = 2\cos^2(A) - 1}$

$2(2\cos^2(x) - 1) = 7\cos(x)$

$4\cos^2(x) - 2 = 7\cos(x)$

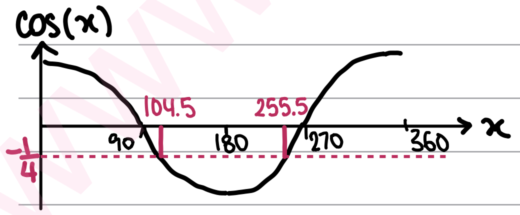
$c = \cos(x)$

$4c^2 - 7c - 2 = 0$

$(4c+1)(c-2) = 0$

$c = -\frac{1}{4}$ ∪ $\cancel{2}$ reject $\cos(x) = 2$ as $-1 \leq \cos(x) \leq 1$

$\therefore \cos(x) = -\frac{1}{4}$



$x = 104.5°, 255.5°$

P 6 5 7 5 8 R A 0 2 3 2

2. A scientist monitored the growth of bacteria on a dish over a 30-day period.

The area, N mm^2, of the dish covered by bacteria, t days after monitoring began, is modelled by the equation

$$\log_{10} N = 0.0646t + 1.478 \qquad 0 \leqslant t \leqslant 30$$

(a) Show that this equation may be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of a to the nearest integer and give the value of b to 3 significant figures.

(4)

(b) Use the model to find the area of the dish covered by bacteria 30 days after monitoring began. Give your answer, in mm^2, to 2 significant figures.

(2)

2. a) $\log_{10} N = 0.0646t + 1.478$

log rules : $\boxed{\log_a b = c \rightarrow a^c = b}$

$N = ab^t$

$10^{(0.0646t + 1.478)} = N$

$N = 10^{0.0646t} \times 10^{1.478}$

$= (30.06...)(10^{0.0646})^t$ a to nearest integer b to 3 s.f.

$= 30(1.16)^t$

b) $N_{30} = 30 \times 1.16^{30}$

$= 2576 \text{ mm}^2 = 2600 \text{ mm}^2$ (2 s.f.)

3.

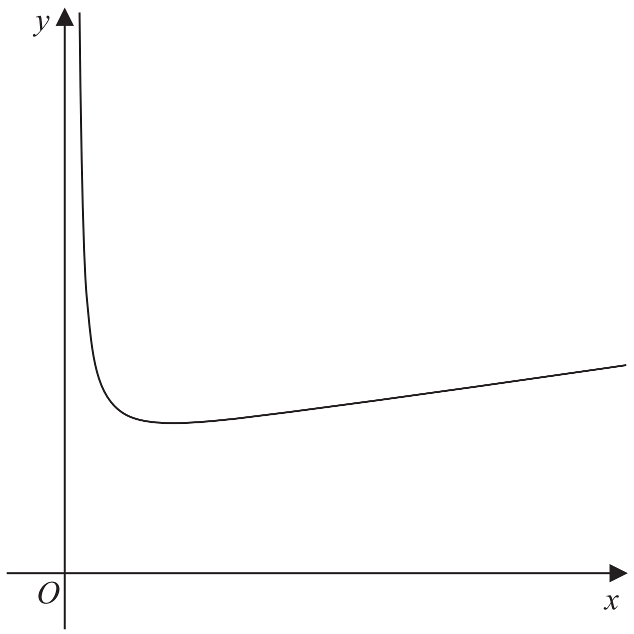


Figure 1

Figure 1 shows a sketch of a curve with equation $y = f(x)$ where

$$f(x) = \frac{2x+3}{\sqrt{4x-1}} \qquad x > \frac{1}{4}$$

(a) Find, in simplest form, $f'(x)$.

(4)

(b) Hence find the range of f.

(3)

3. a) $f(x) = \dfrac{2x+3}{\sqrt{4x-1}}$

Quotient rule for differentiating : $y = \dfrac{u}{v} \rightarrow \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$u = 2x+3 \qquad \dfrac{du}{dx} = 2$

$v = (4x-1)^{\frac{1}{2}} \qquad \dfrac{dv}{dx} = \dfrac{1}{2} \times 4 \times (4x-1)^{-\frac{1}{2}} = 2(4x-1)^{-\frac{1}{2}}$

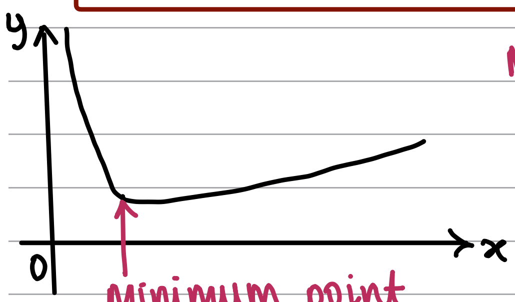
$$f'(x) = \frac{\left((4x-1)^{\frac{1}{2}}\right)(2) - (2x+3)\left(2(4x-1)^{-\frac{1}{2}}\right)}{\left((4x-1)^{\frac{1}{2}}\right)^2}$$

Question 3 continued

multiply through by $(4x-1)^{\frac{1}{2}}$ $\rightarrow$ $= \dfrac{(4x-1)(2) - 2(2x+3)}{(4x-1)^{3/2}}$

$$= \frac{8x - 2 - 4x - 6}{(4x-1)^{3/2}} = \frac{4x-8}{(4x-1)^{3/2}}$$

b) $\boxed{\text{Range} \rightarrow \text{all possible values of } f(x)}$



Minimum is when $f'(x) = 0$:

$$\frac{4x-8}{(4x-1)^{3/2}} = 0$$

$$4x - 8 = 0$$

$$x = 2$$

$\therefore$ min y value

$$y = \frac{2(2)+3}{\sqrt{4(2)-1}} = \frac{7}{\sqrt{7}} = \sqrt{7}$$

$\therefore$ range of $f(x)$ is that $f(x)$ is always bigger than min.

$$f(x) \geq \sqrt{7}$$

4.

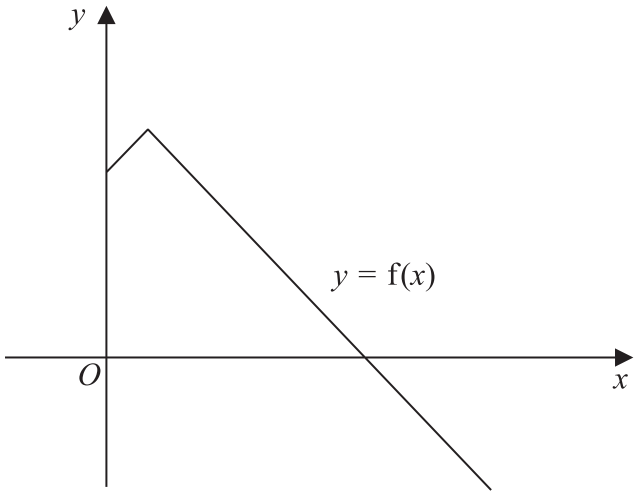


Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 21 - 2|2 - x| \qquad x \geqslant 0$$

(a) Find $\mathrm{ff}(6)$

(2)

(b) Solve the equation $\mathrm{f}(x) = 5x$

(2)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(c) state the set of possible values of k.

(2)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = a\mathrm{f}(x - b)$

The vertex of the graph with equation $y = a\mathrm{f}(x - b)$ is $(6, 3)$.

Given that a and b are constants,

(d) find the value of a and the value of b.

(2)

4.a) $f(x) = 21 - 2|2-x|$

$ff(6) = f(f(6))$

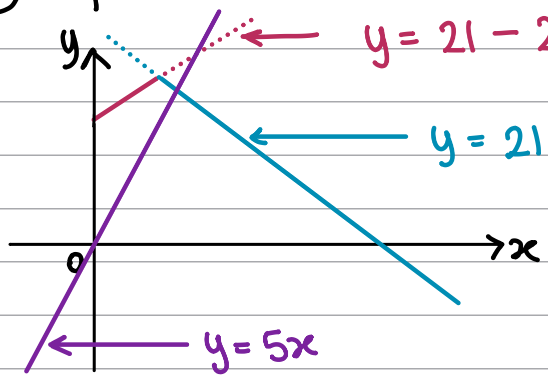
$f(6) = 21 - 2|2-6| \rightarrow f(f(6)) = f(13)$

$= 13 \qquad\qquad = 21 - 2|2-13|$

$= -1$

Question 4 continued

b) $f(x) = 5x$

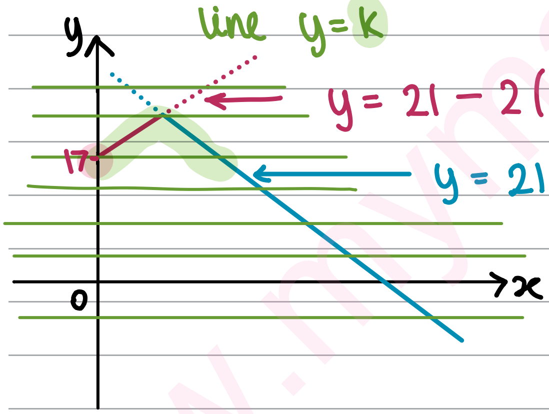


we can see that $y = 5x$ intersects $f(x)$ once – when $y = -2x + 25$

$\therefore \; 5x = -2x + 25$

$7x = 25 \qquad x = \frac{25}{7}$

c) The green lines represent some possibilities of the line $y = k$



For different values of k, there are diff no. of solutions

→ $f(x)$ intersects with the y-axis
↳ $y = 21 - 2|2 - (0)| = 17$

→ max point is intersection between

$2x + 17 = -2x + 25 \qquad \therefore$ max : $(2, 21)$

$4x = 8$

$x = 2$

Question 4 continued

* when $K < 17$ → there is only 1 intersection with part of $f(x)$

* when $17 < K < 21$
↳ $y = K$ intersects with both parts of $f(x)$ ∴ 2 points of intersection

* when $K > 21$
↳ $y = K$ goes above the max point ∴ there are no points of intersection

$\therefore 17 \leq K < 21$

at $K = 21$ there is exactly 1 point of intersection at the max point

at $K = 17$ there are 2 points

d) $y = a f(x - b)$

a: stretch scale factor a parallel to y

$-b$: translation through the vector $\begin{pmatrix} b \\ 0 \end{pmatrix}$

$\therefore (2, 21) \rightarrow (2 + b, a(21))$

$= (6, 3)$

$\therefore b = 4, \; a = \frac{1}{7}$

P 6 5 7 5 8 R A 0 1 2 3 2

5. (a) Show that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x$$

(4)

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$$

(4)

5. a) USING COMPOUND ANGLE FORMULAE → $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$\sin(3x) = \sin(2x + x)$

$\sin(2x+x) = \sin(2x)\cos(x) + \sin(x)\cos(2x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x) = (1-\sin^2(x)) - \sin^2(x)$

$= 1 - 2\sin^2(x)$

$\therefore \sin(3x) = (2\sin(x)\cos(x))\cos(x) + \sin(x)(1-2\sin^2(x))$

$= 2\sin(x)\cos^2(x) + \sin(x) - 2\sin^3(x)$

$= 2\sin(x)(1-\sin^2(x)) + \sin(x) - 2\sin^3(x)$

$= 2\sin(x) - 2\sin^3(x) + \sin(x) - 2\sin^3(x)$

$= 3\sin(x) - 4\sin^3(x)$

Question 5 continued

b) $\int_0^{\frac{\pi}{3}} \sin^3(x)\,dx$

$\sin(3x) \equiv 3\sin(x) - 4\sin^3(x)$

$\sin^3(x) = \dfrac{3\sin(x) - \sin(3x)}{4}$

$\int_0^{\pi/3} \frac{1}{4}\left(3\sin(x) - \sin(3x)\right) dx$

$= \frac{1}{4}\left[-3\cos(x) + \frac{1}{3}\cos(3x)\right]_0^{\frac{\pi}{3}}$

$= \frac{1}{4}\left(-3\cos\left(\frac{\pi}{3}\right) + \frac{1}{3}\cos\left(\frac{\cancel{3}\pi}{\cancel{3}}\right) + 3\cos(0) - \frac{1}{3}\cos(3(0))\right)$

$= \frac{1}{4}\left(-\frac{3}{2} - \frac{1}{3} + 3 - \frac{1}{3}\right) = \frac{1}{4}\left(\frac{5}{6}\right) = \frac{5}{24}$

Q5

(Total 8 marks)

P 6 5 7 5 8 R A 0 1 5 3 2

6.

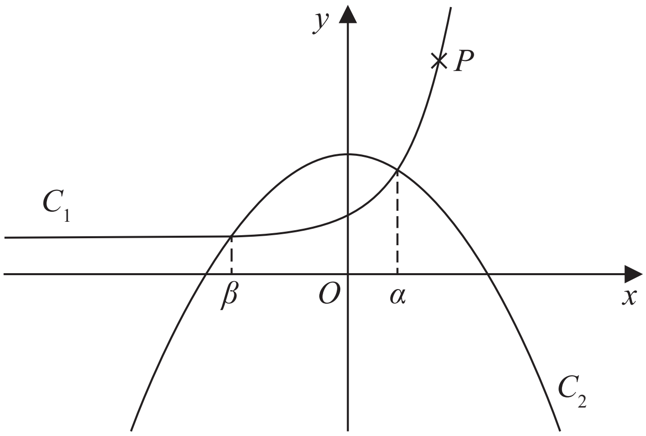


Figure 3

Figure 3 shows a sketch of curve C_1 with equation $y = 5e^{x-1} + 3$

and curve C_2 with equation $y = 10 - x^2$

The point P lies on C_1 and has y coordinate 18

(a) Find the x coordinate of P, writing your answer in the form $\ln k$, where k is a constant to be found.

(3)

The curve C_1 meets the curve C_2 at $x = \alpha$ and at $x = \beta$, as shown in Figure 3.

(b) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places $\alpha = 1.134$

(3)

The iterative equation

$$x_{n+1} = -\sqrt{7 - 5e^{x_n - 1}}$$

is used to find an approximation to β.

Using this iterative formula with $x_1 = -3$

(c) find the value of x_2 and the value of β, giving each answer to 6 decimal places.

(3)

6. a) $C_1 \Rightarrow y = 5e^{x-1} + 3$

P y coordinate $= 18 = 5e^{x-1} + 3 \rightarrow 5e^{x-1} = 15$

$e^{x-1} = 3$

Question 6 continued

$e^{x-1} = 3$

$x - 1 = \ln(3)$

$x = \ln(3) + 1$

LOG RULES → $\log_a b + \log_a c = \log_a(bc)$

$\ln(e) = 1$

$x = \ln(3) + \ln(e)$

$x = \ln(3e)$

b) $C_1 = C_2 \rightarrow 5e^{x-1} + 3 = 10 - x^2$

① set expression to 0 : $7 - x^2 - 5e^{x-1} = 0$

$f(x) = 7 - x^2 - 5e^{x-1}$

To show $\alpha = 1.134$ we have to show there's a sign change

$\left.\begin{array}{l} f(1.1345) = -0.0069 \\ f(1.1335) = 0.0011 \end{array}\right\}$ sign change & given that $f(x)$ is continuous between the range

$\therefore$ the root α lies between 1.1335 & 1.1345

$\therefore$ to 3 d.p. 1.134

c) $x_{n+1} = -\sqrt{7 - 5e^{x_n - 1}}$ $x_1 = -3$

$x_2 = x_{1+1} = -\sqrt{7 - 5e^{x_1 - 1}}$

$= -\sqrt{7 - 5e^{-3-1}} = -\sqrt{7 - 5e^{-4}} = -2.628388$

Turn over

P 6 5 7 5 8 R A 0 1 7 3 2

Question 6 continued

To find the value of β correct to 6 d.p.,

keep using the iterative formula until we get consistent 6. d.p. so write each iteration to 7 d.p.

$x_3 = -2.6205353$

$x_4 = -2.6203355$

$x_5 = -2.6203304$

$x_6 = -2.6203302$

$\therefore \ \beta = -2.620330$

7. (a) Express $\cos x + 4\sin x$ in the form $R\cos(x - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

A scientist is studying the behaviour of seabirds in a colony.

She models the height above sea level, H metres, of one of the birds in the colony by the equation

$$H = \frac{24}{3 + \cos\left(\frac{1}{2}t\right) + 4\sin\left(\frac{1}{2}t\right)} \qquad 0 \leqslant t \leqslant 6.5$$

where t seconds is the time after it leaves the nest.

Find, according to the model,

(b) the minimum height of the seabird above sea level, giving your answer to the nearest cm,

(2)

(c) the value of t, to 2 decimal places, when $H = 10$

(4)

7.a) $\cos x + 4\sin x$ $\qquad R > 0$

$0 < \alpha < \frac{\pi}{2}$

$R\cos(x-\alpha)$ ← using compound angle formulae

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$R(\cos x\cos\alpha + \sin x\sin\alpha)$

Compare expanded expression to given

$R\cos x\cos\alpha + R\sin x\sin\alpha = \cos x + 4\sin x$

$R\cos\alpha = 1 \qquad R\sin\alpha = 4$

$\frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \tan\alpha = \frac{4}{1}$

$\alpha = 1.326$

$(R\cos\alpha)^2 + (R\sin\alpha)^2$ $\qquad \cos^2 A + \sin^2 A = 1$ identity

$= R^2(\cos^2\alpha + \sin^2\alpha) = R^2(1)$

$= 1^2 + 4^2$

$\therefore R^2 = 17 \qquad R = \cancel{\pm}\sqrt{17}$

given that $R > 0$

Question 7 continued

b) $H = \dfrac{24}{3 + \underbrace{\cos\left(\frac{t}{2}\right) + 4\sin\left(\frac{t}{2}\right)}}$ $\quad 0 \leq t \leq 6.5$

$\cos\left(\frac{t}{2}\right) + 4\sin\left(\frac{t}{2}\right) = \sqrt{17}\cos\left(t - 1.326\right)$

$H = \dfrac{24}{3 + \sqrt{17}\cos\left(\frac{t}{2} - 1.326\right)}$

H_{min} will occur when the denominator has its max value

$-1 \leq \cos\left(\frac{t}{2} - 1.326\right) \leq 1$

$-\sqrt{17} \leq \cos\left(\frac{t}{2} - 1.326\right) \leq \sqrt{17}$

$\therefore$ max value of denominator $= 3 + \sqrt{17}$

$H_{min} = \dfrac{24}{3+\sqrt{17}} = 3.37$ m

c) when $H = 10$

$\dfrac{24}{3 + \sqrt{17}\cos\left(\frac{t}{2} - 1.326\right)} = 10$

$24 = 30 + 10\sqrt{17}\cos\left(\frac{t}{2} - 1.326\right)$

$\cos\left(\frac{t}{2} - 1.326\right) = \dfrac{-6}{10\sqrt{17}} = -\dfrac{3\sqrt{17}}{85}$

$t = 2\left(\cos^{-1}\left(-\dfrac{3\sqrt{17}}{85}\right) + 1.326\right) = 6.09$ s

8. (i) The curve C has equation $y = g(x)$ where

$$g(x) = e^{3x} \sec 2x \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

(a) Find $g'(x)$

(2)

(b) Hence find the x coordinate of the stationary point of C.

(3)

(ii) A different curve has equation

$$x = \ln(\sin y) \qquad 0 < y < \frac{\pi}{2}$$

Show that

$$\frac{dy}{dx} = \frac{e^x}{f(x)}$$

where $f(x)$ is a function of e^x that should be found.

(4)

8.(i) a) $g(x) = e^{3x}\sec(2x) \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$

PRODUCT RULE : $y = uv \quad y' = u'v + uv'$

$u = e^{3x} \qquad \frac{du}{dx} = 3e^{3x}$

$v = \sec(2x) \qquad \frac{dv}{dx} = 2\sec(2x)\tan(2x)$

$g'(x) = (3e^{3x})(\sec(2x)) + (e^{3x})(2\sec(2x)\tan(2x))$

$= 3e^{3x}\sec(2x) + 2e^{3x}\sec(2x)\tan(2x)$

b) At stationary points $\rightarrow g'(x) = 0$

$3e^{3x}\sec(2x) + 2e^{3x}\sec(2x)\tan(2x) = 0$

$e^{3x}\sec(2x)(3 + 2\tan(2x)) = 0$

Question 8 continued

$e^{3x} \neq 0 \qquad \sec(2x) \neq 0$

$\therefore \; 3 + 2\tan(2x) = 0$

$\tan(2x) = -\frac{3}{2}$

$x = -0.491 \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$

(ii) $x = \ln(\sin y)$

$\boxed{\text{CHAIN RULE : } \frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}}$

$v = \sin y \qquad \frac{dv}{dy} = \cos y$

$x = \ln(\sin y) = \ln(v) \qquad \frac{dx}{dv} = \frac{1}{v}$

$\frac{dx}{dy} = \frac{dx}{dv} \times \frac{dv}{dy}$

$= \frac{1}{v} \times \cos y = \frac{\cos y}{\sin y}$

$\boxed{\frac{dy}{dx} = \frac{1}{(dx/dy)}}$

$\therefore \frac{dy}{dx} = \frac{1}{\cos y / \sin y}$

$= \frac{\sin y}{\cos y}$

$x = \ln(\sin y)$

$e^x = \sin y$

$\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - e^{2x}}$

$\therefore \frac{dy}{dx} = \frac{\sin y}{\cos y} = \frac{e^x}{\sqrt{1 - e^{2x}}}$

9. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x-4} \qquad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = g(x)$, where

$$g(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \qquad -3 < x < 3.5 \qquad x \in \mathbb{R}$$

(b) Find the equation of the tangent to C at the point where $x = 2$
Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

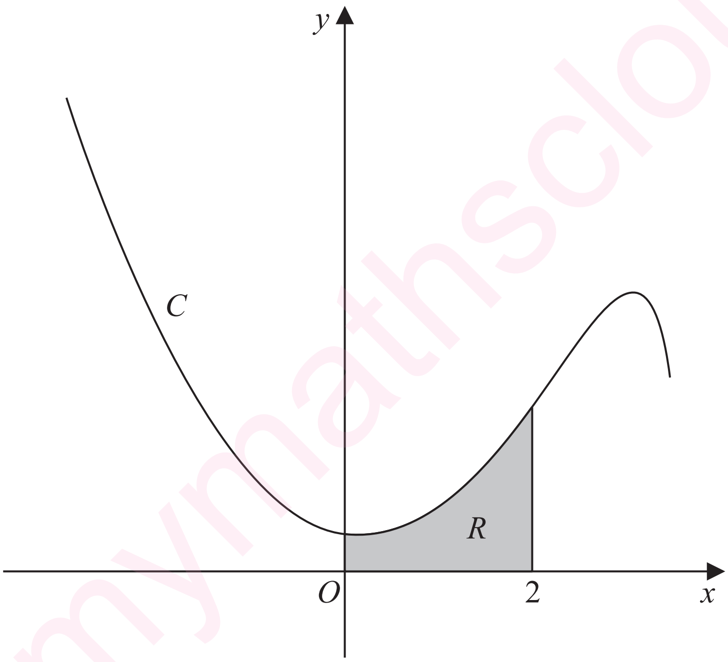


Figure 4

Figure 4 shows a sketch of the curve C.
The region R, shown shaded in Figure 4, is bounded by C, the y-axis, the x-axis and the line with equation $x = 2$

(c) Find the exact area of R, writing your answer in the form $a + b\ln 2$, where a and b are constants to be found.

(5)

9.a) $\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} = x^2 + P + \frac{Q}{x-4}$ $(x > -3)$

Question 9 continued

$$
\begin{array}{r|l}
 & x^2 + 2 \\
x^2-x-12 & x^4 - x^3 - 10x^2 + 3x - 9 \\
 & -\;(x^4 - x^3 - 12x^2) \\
 & 2x^2 + 3x - 9 \\
 & -\;(2x^2 - 2x - 24) \\
 & 5x + 15
\end{array}
$$

$$\therefore \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} = x^2 + 2 + \frac{5x+15}{x^2-x-12}$$

$$= x^2 + 2 + \frac{5\cancel{(x+3)}}{(x-4)\cancel{(x+3)}}$$

$$= x^2 + 2 + \frac{5}{x-4} \qquad [p = 2]$$

b) $g'(x) = \frac{d}{dx}\left(x^2 + 2 + 5(x-4)^{-1}\right)$

$$= 2x - 5(x-4)^{-2}$$

$\therefore$ gradient of C at $x = 2 \rightarrow 2(2) - \frac{5}{(2-4)^2} = \frac{11}{4}$

where $x = 2$, $y = (2)^2 + 2 + \frac{5}{(2)-4} = \frac{7}{2}$

Equation of line : $y - y_1 = m\,(x - x_1)$ coordinates of known point on line

gradient

$$y - \frac{7}{2} = \frac{11}{4}(x - 2)$$

$$y = \frac{11x}{4} - 2$$

Question 9 continued

c) Area under curve $= \int_a^b y \, dx$

$$R = \int_0^2 g(x) \, dx$$

$$= \int_0^2 x^2 + 2 + \frac{5}{x-4} \, dx$$

$$= \left[\frac{x^3}{3} + 2x + 5\ln|x-4| \right]_0^2$$

$$= \frac{2^3}{3} + 2(2) + 5\ln|-2| - 0 - 0 - 5\ln|-4|$$

$$= \frac{20}{3} + 5\ln 2 - 5\ln 4$$

$\boxed{\text{LOG RULES} \rightarrow \quad a\log_b(c) = \log_b(c^a)}$

$$= \frac{20}{3} + 5\ln(2) - 5\ln((2)^2)$$

$$= \frac{20}{3} + 5\ln(2) - 10\ln(2) = \frac{20}{3} - 5\ln(2)$$

P 6 5 7 5 8 R A 0 3 0 3 2

Question 9 continued

(Total 14 marks)

Q9

TOTAL FOR PAPER IS 75 MARKS

END